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TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

23


## CODE:- AG-TS-2-3663

## General Instructions :

1. All question are compulsory.
2. The question paper consists of 26 questions divided into three sections $\mathrm{A}, \mathrm{B}$ and C . Section - A comprises of 6 question of 1 mark each. Section - B comprises of 13 questions of 4 marks each and Section - C comprises of 7 questions of 6 marks each
3. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
4. Use of calculator is not permitted.
5. Please check that this question paper contains 8 printed pages
6. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

## सामान्य निर्देश

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 26 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड - अ में 6 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड - ब में 13 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड - स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 6 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जॉँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 4 हैं।
7. प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।

## PRE-BOARD EXAMINATION 2014-15

| Time: 3 Hours |  | Maximum Marks : 100 |
| :---: | :---: | :---: |
| CLASS - XII | CBSE | MATHEMATICS |
| PART - A |  |  |
| Q. 1 If $\vec{a} \& \vec{b}$ | inclined | s at angles $30^{\circ} \& 120^{\circ}$ |

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|  | respectively, then write the value of $\|\vec{a}+\vec{b}\|$. |
| :---: | :---: |
| Q. 2 | Write the value of $\int_{0}^{\pi / 2} \log \left[\frac{3+5 \cos x}{3+5 \sin x}\right] d x$ |
| Q. 3 | For two non zero vectors $\vec{a}$ and $\vec{b}$ write when $\|\vec{a}+\vec{b}\|=\|\vec{a}\|+\|\vec{b}\|$ holds. |
| Q. 4 | A matrix A of order $3 \times 3$ has determinant 5 . What $s$ the value of 13AI? |
| Q. 5 | Write the smallest equivalence relation R on Set $\mathrm{A}=\{1,2,3\}$ |
| Q. 6 | A four digit number is formed using the digits $1,2,3,5$ with no repetitions. Find the probability that the numbers is divisible by 5 . |
|  | PART - B |
| Q. 7 | Evaluate : $\int \frac{2+\sin x}{1+\cos x} \cdot e^{x / 2} \cdot d x$ <br> OR <br> Evaluate : $\int \frac{5 x}{(x+1)\left(x^{2}+9\right)} d x$. |
| Q. 8 | A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi - vertical angle is tan ${ }^{1}(1 / 2)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10 m . |
| Q. 9 | If $A=\left[\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right]$, prove that $(a I+b A)^{n}=a^{n} . I+n a^{n-1} b A$ where $I$ is a unit matrix of order 2 and n is a positive integer. <br> OR |

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If $\mathrm{a}, \mathrm{b}$ and c are real numbers and $\left|\begin{array}{lll}b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a\end{array}\right|=0$. Show
that either $\mathrm{a}+\mathrm{b}+\mathrm{c}=0$ or $\mathrm{a}=\mathrm{b}=\mathrm{c}$.
Q. 10

Show that the function $y=(A+B x) e^{3 x}$ is a solution of the equation $\frac{d^{2} y}{d x^{2}}-6 \frac{d y}{d x}+9 y=0$.
Q. 11 Find the shortest distance between the lines, whose equations are $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{10-z}{-7}$ and $\frac{x-15}{3}=\frac{58-2 y}{-16}=\frac{z-5}{-5}$. Also find the angle between two lines.

## OR

Find the equation of the plane passing through the intersection of the planes, $2 \mathrm{x}+3 \mathrm{y}-\mathrm{z}+1=0$; $\mathrm{x}+\mathrm{y}-2 \mathrm{z}+3=0$ and perpendicular the plane $3 x-y-2 z-4=0$. also the inclination of this plane with the xy-plane.
Q. 12 Show that the differential equations $2 y e^{x / y} d x+\left(y-2 x e^{x / y}\right) d y=0$ is homogeneous and find its particular solution given that $\mathrm{x}=0$ when y $=1$.

## OR

The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?
If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} \cdot \vec{b}=\vec{a} \cdot \vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a}= \pm 2(\vec{b} \times \vec{c})$.

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| Q. 14 | A man is known to speak truth 5 out of 6 times. He draws a ball from the bag containing 4 white and 6 black balls and reports that it is white. Find the probability that it is actually white? Do you think that speaking truth is always good? |
| :---: | :---: |
| Q. 15 | If $y=\sin ^{-1}\left(\sqrt{x^{4}-x^{6}}+\sqrt{x^{2}-x^{6}}\right)$ Prove that $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{2 \mathrm{x}}{\sqrt{1-\mathrm{x}^{4}}}+\frac{1}{\sqrt{1-\mathrm{x}^{2}}}$. |
| Q. 16 | If $\sin ^{-1} \frac{2 p}{1+p^{2}}-\cos ^{-1} \frac{1-q^{2}}{1+q^{2}}=\tan ^{-1} \frac{2 x}{1-x^{2}}$ then prove that $\mathrm{x}=\frac{p-q}{1+p q}$. |
| Q. 17 | Evaluate : $\int_{1}^{3}\left(5 x^{2}-e^{x}+4\right) d x$ as a limit of sums |
| Q. | Discuss the continuity and differentiability of $\mathrm{f}(\mathrm{x})=$ $\left\{\begin{array}{cc}1-x & x<1 \\ (1-x)(2-x) & 1 \leq x \leq 2 \\ 3-x & x>2\end{array} \quad\right.$. at $\mathrm{X}=1 \& \mathrm{X}=2$. |
| Q. 19 | Evaluate $: \int_{0}^{\pi} \frac{x d x}{1-\cos \alpha \sin x}$ |
|  | PART-C |
| Q. 20 | Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. It is found that the shopkeepers A, B, C are using $(20,30,40),(30,40$, $20),(40,20,30)$ polythene, handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spent Rs. 250 , Rs. 270\& Rs. 200 on these carry bags respectively. Find the cost of each carry bags using matrices. Keeping in the mind the social \& environmental conditions, which shopkeeper is better? Why? |
| Q. 21 | Find the probability that a year chosen at random has 53 Sundays. |
| Q. 22 | Find the area of the origin : $\left\{(x, y): 0 \leq y \leq x^{2}, 0 \leq y \leq x+2 ; 0 \leq x \leq 3\right\}$ |

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OR
Find the ratio of the areas into which curve $y^{2}=6 x$ divides the region bounded by $\mathrm{x}^{2}+\mathrm{y}^{2}=16$.
Q. 23 An isosceles triangle of vertical angle $2 \theta$ is inscribed in a circle of radius $a$. Show that the area of triangle is maximum when $\theta=\frac{\pi}{6}$. OR
A point on the hypotenuse of a right triangle is at a distance ' $a$ ' and ' $b$ ' from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left[\mathrm{a}^{2 / 3}+\mathrm{b}^{2 / 3}\right]^{3 / 2}$.
Q. 24 Avinash has been given two lists of problems from his mathematics teacher with the instruction to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. He knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way, how many problems from each list shall he do to maximize his marks? What is the importance of time management for students?
Q. 25 Find the foot of the perpendicular from $\mathrm{P}(1,2,3)$ on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$.Also obtain the equation of the plane containing the line and the point $(1,2,3)$.
Q. 26 Let X be a non - empty set. P(x) be its power set. Let ** be an operation defined on element of $\mathrm{P}(\mathrm{x})$ by $\mathrm{A} * \mathrm{~B}=\mathrm{A} \cap \mathrm{B} \forall \mathrm{A}, \mathrm{B} \in$ $\mathrm{P}(\mathrm{X})$ Then,
(i) Prove that * is a binary operation in $\mathrm{P}(\mathrm{X})$.

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(ii) Is* commutative ?
(iii) Is* associative?
(iv)find the identity element in $\mathrm{P}(\mathrm{X})$ w.r.t * .
(v) find the all the invertible element of $\mathrm{P}(\mathrm{X})$
(vi) if O is another binary operation defined on $\mathrm{P}(\mathrm{X})$ as $\mathrm{A} \mathrm{OB}=\mathrm{A} \cup$ $B$ then verify that O distribution itself over *.

NOBODY CAN BE SUCCESSFUL IF HE DOESN'T LOVE HIS WORK

